# Exercise 38

According to Boyle's Law, if the temperature of a confined gas is held fixed, then the product of the pressure $P$ and the volume $V$ is a constant. Suppose that, for a certain gas, $PV = 800$, where $P$ is measured in pounds per square inch and $V$ is measured in cubic inches.

(a) Find the average rate of change of $P$ as $V$ increases from 200 $\text{in}^3$ to 250 $\text{in}^3$.

(b) Express $V$ as a function of $P$ and show that the instantaneous rate of change of $V$ with respect to $P$ is inversely proportional to the square of $P$.

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**Solution**

## Part (a)

Solve the provided equation of state for $P$.

$$P = \frac{800}{V}$$

The average rate of change of $P$ as $V$ increases from 200 $\text{in}^3$ to 250 $\text{in}^3$ is given by the slope of the secant line.

$$m = \frac{P(250) - P(200)}{250 - 200} = \frac{\left(\frac{800}{250}\right) - \left(\frac{800}{200}\right)}{50} = -0.016$$

The units of this rate of change are $(\text{lb}/\text{in}^2)/\text{in}^3$, or $\text{lb}/\text{in}^5$.

## Part (b)

Solve the provided equation of state for $V$.

$$V = \frac{800}{P}$$

Use the definition of the derivative.

$$V'(P) = \lim_{h \to 0} \frac{V(P+h) - V(P)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{800}{P+h} - \frac{800}{P}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{800P}{P(P+h)} - \frac{800(P+h)}{P(P+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{800P - 800(P+h)}{P(P+h)}}{h}$$

$$= \lim_{h \to 0} \frac{800P - 800(P+h)}{hP(P+h)}$$

Cancel out $h$ and evaluate the limit.

$$\begin{aligned} V'(P) &= \lim_{h \to 0} \frac{800P - 800P - 800h}{hP(P+h)} \\ &= \lim_{h \to 0} \frac{-800h}{hP(P+h)} \\ &= \lim_{h \to 0} \frac{-800}{P(P+h)} \\ &= \frac{-800}{P(P+0)} \\ &= -\frac{800}{P^2} \end{aligned}$$

Therefore, the instantaneous rate of change of $V$ with respect to $P$ is inversely proportional to the square of $P$.